Low Frequency Modifications of Seismic Background Noise Due to Interaction with Oscillating Fluids in Porous Rocks

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SUMMARY

Low frequency spectral modifications of seismic background waves (noise) due to interaction with partially saturated porous rocks are investigated. Non-wetting fluid drops entrapped in pores can oscillate with a characteristic eigenfrequency. A 1D wave equation is coupled with a linear oscillator equation representing these oscillations. The resulting system of equations is solved numerically with explicit finite differences. The background noise is reduced to its dominant frequency (0.1-0.3Hz) which is presumably related to surface waves generated by ocean waves. This frequency is used as the external source. The resulting incident monochromatic wave excites the pore fluid which thereafter oscillates with its eigenfrequency. Oscillatory energy is transferred to the porous rock which leads to an amplitude decay of the fluid oscillation. The elastic matrix carries the eigenfrequency of the fluid oscillation in addition to the external frequency. Fourier spectra of the solid velocity therefore show two distinct peaks: the external frequency and the eigenfrequency of the fluid oscillation. Interestingly, such low frequency modifications of seismic noise are observed above hydrocarbon reservoirs and the presented model is considered as one possible explanation. Time evolution of the amplitude decay of the fluid oscillation seems to be related to the thickness of the porous rock.
Introduction

The behavior of fluids entrapped in a capillary tube and in idealized pores were thoroughly studied in the past (e.g. Graham and Higdon 2000a,b or Dvorkin et al. 1990). In hydrocarbon industry the results of these studies were used to develop a new enhanced oil recovery technology (EOR) which is termed wave stimulation of oil production or vibratory mobilization (Beresnev and Johnson 1994). The basic idea is to excite oscillations of the entrapped oil which are strong enough for the fluid to eventually leave the pore. The present work does not focus on the artificially induced pore fluid oscillations but on the naturally occurring oscillations generated by the ever present seismic background noise. A new model is presented that couples an oscillation equation with a 1D elastic wave propagation equation.

Coupling between pore fluid oscillations and elastic waves

Non-wetting fluid drops entrapped in a porous rock can oscillate within the pores with a characteristic eigenfrequency $\omega_0$ (Hilpert et al. 2006, Beresnev 2006). Capillary forces act as restoring forces that drive the oscillations. For idealized pore geometry this oscillation can be approximated by a second order ordinary differential equation.

$$\frac{\partial^2 u_f}{\partial t^2} = -\omega_0^2 u_f$$ (1)

Where $t$ is time, $u_f$ the displacement of the fluid out of its equilibrium position and $\omega_0$ the eigenfrequency of the linear oscillation, which is calculated from material parameters describing the pore geometry and the fluid (e.g. surface tension and density). For frequently occurring physical values, $\omega_0$ is in the low frequency range (Holzner et al. 2006). Wave propagation in the elastic porous rock in one dimension is described by a second order partial differential equation.

$$\frac{\partial^2 u_s}{\partial t^2} = \frac{1}{(1-\phi)\rho_s} \frac{\partial}{\partial x} \left(E \frac{\partial u_s}{\partial x}\right) + \frac{1}{(1-\phi)\rho_s} F(x,t)$$ (2)

Where $u_s$ is the displacement of the solid, $\phi$ the porosity of the rock, $\rho_s$ the density of the rock material, $E$ the Young’s modulus and $x$ the spatial coordinate. $F$ represents any external force acting on the elastic porous rock. To couple equation (1) and (2) both are written in terms of volumetric forces. The restoring force term has to be written in terms of relative displacements and is treated as an addition force term acting on the elastic solid.

$$\phi \rho_f \frac{\partial^2 u_f}{\partial t^2} = -\phi \rho_f \omega_0^2 (u_f - u_s)$$

$$(1-\phi) \rho_s \frac{\partial^2 u_s}{\partial t^2} = \frac{\partial}{\partial x} \left(E \frac{\partial u_s}{\partial x}\right) + \phi \rho_f \omega_0^2 (u_f - u_s) + F(x,t)$$ (3)

This system of equations is solved numerically using explicit finite differences on a staggered grid (Virieux 1986). The applied physical parameters are given in Table 1.

Energy conservation and transfer

In the introduced model four different volumetric energies can be calculated and integrated over the model domain.

$$E_f^{\text{kin}} = \int_{\text{domain}} \frac{\phi \rho_f}{2} v_f^2 \text{d}x, \quad E_f^{\text{el}} = \int_{\text{domain}} \frac{\phi \rho_f}{2} \omega_0^2 (u_f - u_s)^2 \text{d}x$$

$$E_s^{\text{kin}} = \int_{\text{domain}} \frac{(1-\phi) \rho_s}{2} v_s^2 \text{d}x, \quad E_s^{\text{el}} = \int_{\text{domain}} \frac{E}{2} \left(\frac{\partial u_s}{\partial x}\right)^2 \text{d}x$$ (4)

Where $E_f^{\text{kin}}$ and $E_s^{\text{kin}}$ are kinetic and elastic (strain) energies of the fluid (subscript $f$) and the solid (subscript $s$), respectively. Using two reflecting boundary conditions and omitting the external force, the total energy in the model domain, i.e. the sum of all four energies stays
constant over time. In the simulation shown (Figure 1) a model size of 420m and a Gaussian initial condition for the solid velocity was applied. The initial condition influences the time evolution of the different energies in that sense that at the initiation of the simulation only kinetic energy of the solid is present. After that, the energy is distributed between solid and fluid and transferred back and forth between the two media (thick black and thick red line). This transfer happens in the same order of magnitude as the total energy of the system. This suggests that the energy transfer from the pore fluid oscillations to the solid elastic medium is strong enough to influence the spectral content of the elastic waves.

Table 1: Parameters used in numerical simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenfrequency of pore fluid oscillation</td>
<td>( \omega_0 )</td>
<td>18.85 (= 3 Hz ( \cdot ) 2( \pi ))</td>
</tr>
<tr>
<td>Density of fluid</td>
<td>( \rho_f )</td>
<td>800 kg m(^{-3} )</td>
</tr>
<tr>
<td>Density of solid</td>
<td>( \rho_s )</td>
<td>2800 kg m(^{-3} )</td>
</tr>
<tr>
<td>Youngs modulus</td>
<td>( E )</td>
<td>( 2 \cdot 10^{10} ) Pa</td>
</tr>
<tr>
<td>Porosity</td>
<td>( \phi )</td>
<td>0.3</td>
</tr>
<tr>
<td>External frequency</td>
<td>( \Omega )</td>
<td>1.89 (= 0.3 Hz ( \cdot ) 2( \pi ))</td>
</tr>
<tr>
<td>Total time</td>
<td>( T_{\text{sw}} )</td>
<td>120s</td>
</tr>
</tbody>
</table>

Figure 1: Time evolution of different energies in the model domain. The total energy, i.e. the sum of all energies (green line) stays constant over time.

External source and model setup

The Fourier spectrum of a typical measurement of ambient seismic background waves (noise) shows a very distinct peak at around 0.1-0.3Hz (left gray bar in Figure 2). This high energy spectral peak is a global feature that can be measured everywhere in the world. It is presumably related to seismic surface waves generated by ocean waves (e.g. Aki and Richards 1980). In this study the seismic background noise is reduced to this dominant peak. The external source term in equation (3) becomes

\[
F(x,t) = A_s(x) \sin(\Omega t) \tag{5}
\]

with

\[
A_s(x) = \begin{cases} 
0 & \text{for } x \neq x_{\text{source}} \\
1 & \text{for } x = x_{\text{source}} 
\end{cases} \tag{6}
\]

where \( \Omega = 1.89 \) (= 0.3Hz \( \cdot \) 2\( \pi \)).

Figure 2: Field measurements of ambient seismic background noise. One measurement above (red) and one nearby (blue) the location of a producing subsurface hydrocarbon reservoir. Spectraseis campaign: Mososro, Mexico, 2002

Figure 3: Model setup for numerical simulations consists of two non-reflection boundary conditions, three receivers \( R_1-R_3 \) and one source \( S \).
The external source is applied only at one point $x_{\text{source}}$ in the model domain. The model setup (Figure 3) for the following numerical simulations consists of two non-reflecting boundary conditions (Ionescu and Igel 2003), three receivers and the point source described above. The position of the source is identical to one of the receivers.

**Numerical results**

Numerical simulations were run for 120s physical time. Fourier transforms of the recorded solid velocity time signals were calculated for all three receivers (Figure 4). They all show a very distinct peak at the frequency of the external force at 0.3Hz, as expected. In addition a second peak appears in all spectra. It coincides with the eigenfrequency of the pore fluid oscillations $\omega_0$. The excited pore fluid oscillations are transferred to the elastic porous matrix. This transfer is strong enough that the corresponding frequency, the eigenfrequency of the oscillations is carried on top of the externally applied frequency $\Omega$. Both signals can be measured in the solid velocity signal at any point of the model domain. The trough of the spectra at receiver R1 (blue line in Figure 4) is an artifact of the fast Fourier transform (FFT) and has no physical meaning.

The pore fluid oscillations are initiated by the incident monochromatic wave generated by the external force. While the wave itself is monochromatic, the wave front consists of all frequencies. Therefore the pore fluid starts to oscillate with its eigenfrequency. After the wave front has passed, the only externally applied frequency is $\Omega$. Its amplitude stays constant over time. All other frequencies decay in amplitude over time (Figure 5).

![Figure 4: Fourier transforms of solid velocity time signals for three receivers R1-R3 (Figure 3). Positions of receivers are measured from top of model domain. Dashed line: Frequency of external force (0.3Hz); Solid line: Eigenfrequency of pore fluid oscillations (3Hz)](image)

![Figure 5: Spectra of solid velocity measured at receiver R1 (Figure 3) after different amount of time. Longest time signal is 120s, shortest is 3.5s. Black vertical line: Frequency of external force; Red vertical line: Eigenfrequency of pore fluid oscillations; Green vertical line: Frequency with minimum power from 1 to 3Hz = 2Hz](image)

![Figure 6: Spectral amplitude decay over time at the eigenfrequency of pore fluid oscillations (3Hz) for different porous layer thicknesses. This amplitude is normalized with the amplitude at the frequency of the external source (0.3Hz), which stays constant over time. The solid velocity to calculate this is measured outside the porous layer.](image)
The pore fluid continues to oscillate with its eigenfrequency and constantly transfers energy to the elastic porous rock. Although this frequency also decays in amplitude over time it is relatively higher in the spectrum (Figure 5).

A second set of numerical simulations were performed. The homogeneous model setup was expanded with two purely elastic layers on top and at the bottom of the model. The external point source was applied below the porous layer and an additional receiver was added on top, both 7m away from the interface between porous and purely elastic layer. Several runs with a different thickness of the porous layer were performed. The solid velocity at the newly added receiver was recorded and the Fourier spectra calculated. As in the homogeneous model (Figure 5) the amplitude at the eigenfrequency of the pore fluid oscillation is elevated and it decays with time. Interestingly, this decay over time is different for different thicknesses of the porous layer (Figure 6). A thick porous layer creates higher amplitudes which decay linearly with time in a log-log-diagram. A thin porous layer creates smaller amplitudes, but the decay is slower until the amplitude asymptotically reaches the values for thicker layers. A saturation of this effect occurs at around 70m thickness of the porous layer.

Discussion and conclusions

Oscillations of entrapped fluid droplets in a porous elastic rock can be transferred to the solid. This transfer is strong enough that the additional frequency, the eigenfrequency of the pore fluid oscillations is visible in the Fourier spectrum of the solid velocity. Modifications of seismic background noise in the low frequency range were observed in nature above hydrocarbon bearing structures (Figure 2, right gray bar; Dangel et al. 2003). The physical explanation for these modifications is the subject of discussion (Graf et al. 2007). Among other phenomena, such as seismic attenuation, reflections or scattering, oscillations of pore fluids have to be considered as one possible explanation. First results suggest that thickness information of hydrocarbon bearing structures can be extracted from seismic background noise measurements.

References