Theoretical and numerical modeling of waves in three-phase media
-a snapshot of the work in progress -

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Abstract
We present a mathematical model for wave propagation in a partially saturated porous medium based on the Theory of Porous Media (TPM) [1, 10]. The medium is composed of a elastic skeleton, a compressible wetting fluid and a compressible non-wetting fluid. The solid grains are assumed incompressible (rigid grain assumption). The capillary pressure depending on saturation is described by the empirical Brooks and Corey equation [8]. Both porosity and saturation depend on the volumetric deformation of all phases and are variable (i.e. they are dependent field variables).

We developed a new three-phase model for wave propagation in partially-saturated porous media with gas or air as third phase. The restoring force due to surface tension can cause oscillations of water or oil blobs in partially-saturated porous material with gas or air as third phase. The restoring force due to surface tension is usually not considered in continuum three-phase models for wave propagation.

The main aims of our study are:
•Coupling wave propagation in partially saturated rocks with pore fluid oscillations caused by surface tension.
•Implementation of attenuation due to fluid oscillations.
•Study attenuation of P- and S-waves in three-phase media due to wave induced flow (e.g., depth dependence to gas pressure increase).
•Accurate implementation of capillary pressure for cases of very high or low saturation.
•Study scattering of waves by three-phase media.

•Develop a simpler but still accurate three-phase model for partially gas-saturated rocks
•Accurate implementation of capillary pressure for cases of very high or low saturation.

The main results of this work are:

1. A mathematical model for wave propagation in partially saturated porous media based on the Theory of Porous Media (TPM) [1, 10]. The medium is composed of an elastic skeleton, a compressible wetting fluid and a compressible non-wetting fluid. The solid grains are assumed incompressible (rigid grain assumption). The capillary pressure depending on saturation is described by the empirical Brooks and Corey equation [8]. Both porosity and saturation depend on the volumetric deformation of all phases and are variable (i.e. they are dependent field variables).

2. We calculated the three P-wave phase velocities (P1, P2, P3) and the corresponding attenuation coefficients from the eigenvalues of the governing system of equations. The results of the Gassmann-Wood limit (e.g. [2]) were calculated for a value of the bulk modulus of the solid grain of 35 GPa. The first P-wave (P1) phase velocity of the three-phase model assuming incompressible grains agrees well with the Gassmann-Wood limit assuming compressible grains. The results are calculated for a Sandstone partially saturated with either air or water and gas and water.

3. Results of accuracy tests for different numerical algorithms. The analytical solution is compared with the finite element mesh. The Biot equations [9] have been implemented using a velocity-stress formulation.

4. The way forward to numerical implementation. The analytical solution is compared with the finite element mesh. The Biot equations [9] have been implemented using a velocity-stress formulation.

5. The main result: The finite element method with explicit time integration yields best results with respect to accuracy and computation time.

References:

Some basic equations of the mathematical model with effective surface tension term

Field equations

\[ \rho \ddot{u} + \gamma \div \nabla \ddot{u} = - \rho \alpha \gamma \nabla \phi \div \nabla \phi \]  
\[ \rho \ddot{u} + \gamma \nabla \div \nabla \alpha \ddot{u} = - \rho \alpha \gamma \phi \nabla \div \nabla \phi \]

Constitutive relations

\[ \sigma_{ij} = 2\mu \epsilon_{ij} + \lambda \epsilon_{ii} \delta_{ij} \]

\[ S = \frac{\gamma}{\alpha} \frac{\nabla \phi \nabla \phi}{\gamma} \]

\[ \alpha = \frac{1}{1 + \gamma \phi} \]

Results of the three-phase model (so far without surface tension)

We compared the finite difference and finite element methods for modeling two-dimensional (2D) scattering of elastic waves by a weak circular inclusion [11]. Time integration has been done with both explicit and implicit methods, also with an implicit finite element time integration. We compared the numerical results with an analytical solution for 2D scattering of elastic waves based on displacement potentials using Hankel functions [7]. The finite element method has been used to solve Biot's equation [9] and will be used to solve the above described TPM equations.

The way forward to numerical implementation

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Main result: The finite element method with explicit time integration yields best results with respect to accuracy and computation time.