Phase velocity dispersion and attenuation of seismic waves due to trapped fluids in residual-saturated porous media

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Abstract

To model seismic wave propagation in residual saturated porous media, we developed a three-phase model based on a continuum mixture theory capturing the coupling between the micro- and the macroscale. The model considers a continuous and a discontinuous part. The continuous part is equivalent to the poroelastic Biot-model. The discontinuous part describes the movement of blobs/clusters of the residual wetting fluid by an oscillator rheology. The presented model accounts for the heterogeneity of the discontinuous fluid clusters by a model-embedded distribution function of the cluster sizes. We define a dimensionless parameter determining if the motion of the residual fluid is dominated by oscillations (underdamped, resonance) or not (overdamped). Our results show that the residual fluid has a significant impact on the velocity dispersion and attenuation no matter if it oscillates or not. We show under which conditions and how the classical Biot-model can be used to approximate the dynamic behavior of residual saturated porous media.

The model

Phase 1 & 2: Biot medium (Figure 1, blue)

- Two phases: Solid skeleton & continuous non-wetting fluid
- 2-phase mixture theory (Biot-theory; Biot, 1956)
- Coupled system of PDE's
- 2 P-waves, 1 S-wave

Phase 3: Discontinuous wetting fluid (Figure 1, orange)

- No propagating wave because it is discontinuous
- But relative displacement (elastic) between trapped viscous wetting fluid blobs/clusters and solid skeleton
- Therefore, damped oscillator rheology for wetting fluid
- Each blob/cluster size exhibits a different resonance frequency. Therefore, a damped harmonic oscillator equation is defined for each size.

Coupling yields 3-phase residual saturation model

- Momentum exchange between Biot medium and each oscillating fluid blob/cluster:

$$N\nabla\cdot\nabla\mathbf{u}_s+(A+N)\nabla\nabla\cdot\mathbf{u}_s+S\nabla\nabla\cdot\mathbf{u}_n=\rho_s\ddot{\mathbf{u}}_s-b_0(\dot{\mathbf{u}}_n-\dot{\mathbf{u}}_s)-\sum_{k=1}^{z}\left[\alpha_k\rho_w\omega_k^2(\mathbf{u}_{w,k}-\mathbf{u}_s)+\alpha_k n_w c_k(\dot{\mathbf{u}}_{w,k}-\dot{\mathbf{u}}_s)\right]$$

$$S\nabla\nabla\cdot\mathbf{u}_s+R\nabla\nabla\cdot\mathbf{u}_n=\rho_n\ddot{\mathbf{u}}_n+b_0(\dot{\mathbf{u}}_n-\dot{\mathbf{u}}_s)$$

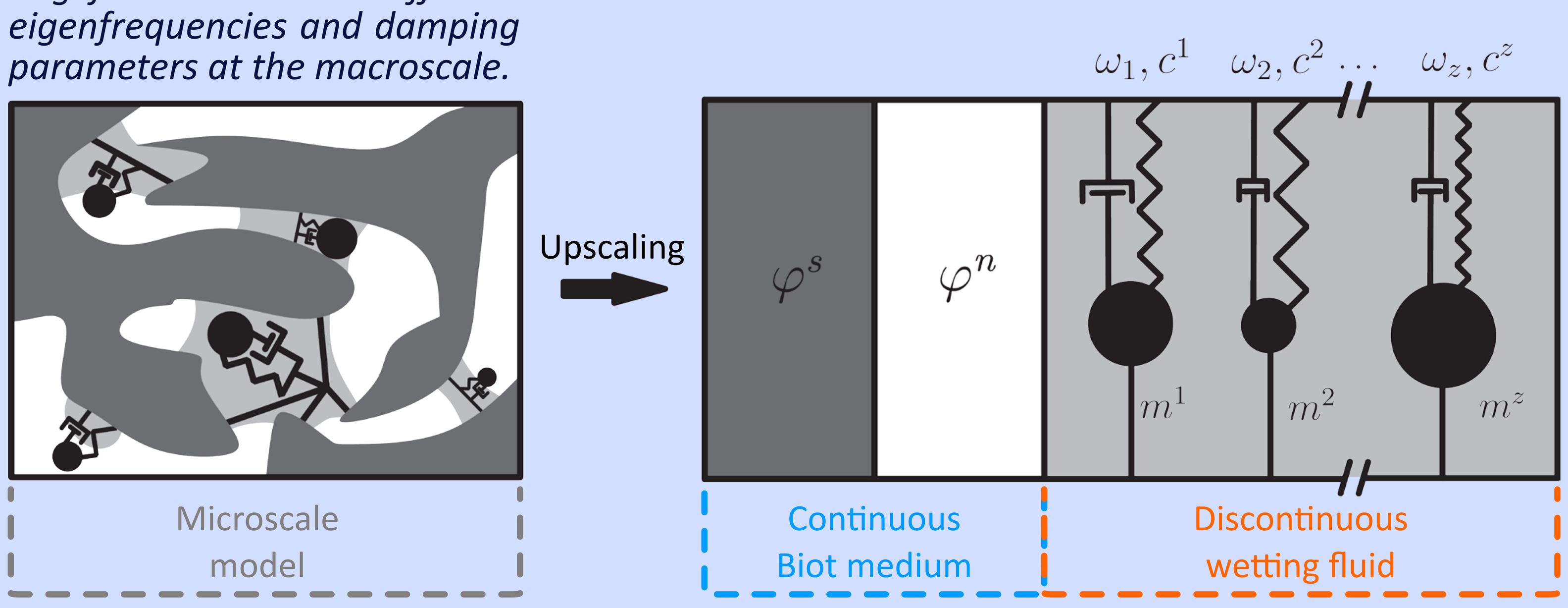
$$\mathbf{0}=\rho_w\ddot{\mathbf{u}}_{w,1}+\rho_w\omega_1^2(\mathbf{u}_{w,1}-\mathbf{u}_s)+n_w c_1(\dot{\mathbf{u}}_{w,1}-\dot{\mathbf{u}}_s)$$

$$\vdots$$

$$\mathbf{0}=\rho_w\ddot{\mathbf{u}}_{w,z}+\rho_w\omega_z^2(\mathbf{u}_{w,z}-\mathbf{u}_s)+n_w c_z(\dot{\mathbf{u}}_{w,z}-\dot{\mathbf{u}}_s)$$

Momentum exchange

Figure 1: Upscaling from the heterogeneous pore scale to an oscillatoric behavior of the wetting fluid blobs with different eigenfrequencies and damping parameters at the macroscale.



Continuous fluid blob/cluster size distribution

- Above equations are for discrete distribution of z fluid blob sizes.
- More realistic is a continuous blob/cluster size distribution, $\alpha(\omega)$.
- This is achieved by a simple transistion from sum to integral: $\Sigma\rightarrow\int$

Model results

Single oscillator

- Only one size of fluid blobs/clusters, i.e. only one resonance frequency
- Different damping parameters, D

$$D=\frac{n_w c_1}{\rho_w\omega_1}=\frac{c_1}{\rho_{wR}\omega_1}$$

D is the ratio between viscous damping and oscillation.

- *Strong damping* of oscillators ($\log_{10}(D)>0.5$): Phase velocity and attenuation are identical to that of the Biot-theory (Figure 2), but shifted to higher frequencies.
- *Weak damping* of oscillators ($\log_{10}(D)<-0.1$): Phase velocity and attenuation exhibit strong anomalies at the resonance frequency.

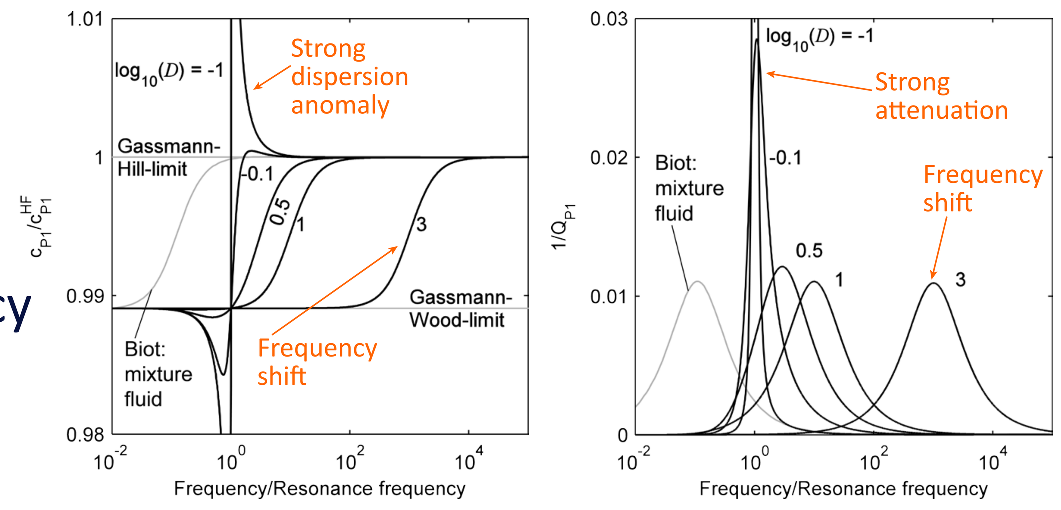


Figure 2: Phase velocity dispersion (left) and frequency-dependent attenuation of the P_1-wave for different damping parameters, D. The porous skeleton is a Berea sandstone saturated with a continuous gas phase.

Continuous distribution of fluid blob/cluster sizes

- Various sizes of fluid blobs/clusters, i.e. continuous distribution of resonance frequencies described as probability density function
- Log-normal size distribution with different widths (Figure 3a; parameter s).
- Stronger damping of oscillations for smaller fluid blobs (Figure 3a).
- *Narrow size distribution* ($s<1$): Phase velocity and attenuation resemble single-osicllator case with small D, i.e. strong anomalies.
- *Wide size distribution* ($s>2$): Phase velocity and attenuation resemble single-oscillator case with large D, i.e. identical to Biot-theory but shifted to higher frequencies.

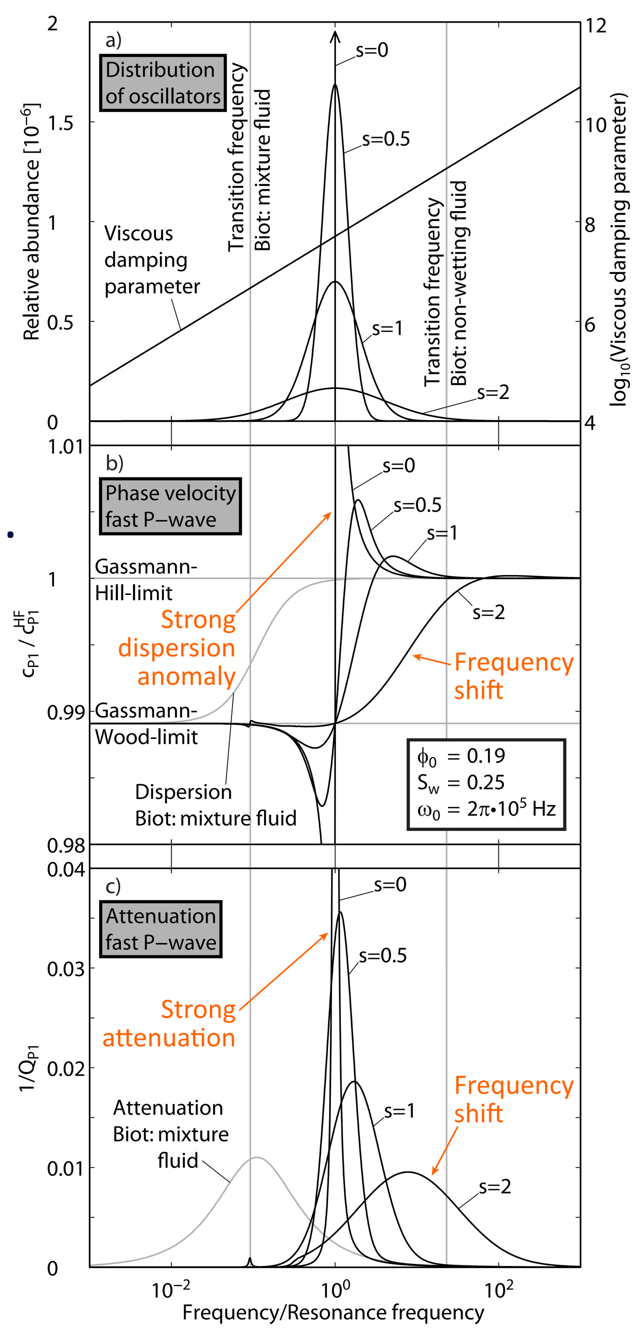


Figure 3: Phase velocity dispersion (b) and frequency-dependent attenuation (c) of the P_1-wave for different size distributions of fluid blobs/clusters, s (a). The porous skeleton is a Berea sandstone saturated with a continuous gas phase.

Implications / Conclusions

Trapped fluid blobs/clusters of a residual saturated wetting fluid strongly influence the velocity dispersion and attenuation, no matter if they oscillate or not.

For an oscillating trapped fluid, the dispersion and attenuation show distinct anomalies. The oscillations can store energy and release it again over time (Frehner et al., 2009, 2012; Steeb et al., 2010, 2012).

For damped oscillations, the dispersion and attenuation are identical to the ones for the Biot-theory (Biot, 1956) that is shifted to higher frequencies. In this case, the dispersion and attenuation can be described by an effective two-phase Biot-theory (Steeb et al., 2012).

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